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British Mathematical Olympiad

Round 1 : Wednesday, 3 December 2003

Time allowed *Three and a half hours.*

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

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2003/4 British Mathematical Olympiad
Round 1

1. Solve the simultaneous equations

$$ab + c + d = 3, \quad bc + d + a = 5, \quad cd + a + b = 2, \quad da + b + c = 6,$$

where a, b, c, d are real numbers.

2. $ABCD$ is a rectangle, P is the midpoint of AB, and Q is the point on PD such that CQ is perpendicular to PD.
Prove that the triangle BQC is isosceles.

3. Alice and Barbara play a game with a pack of $2n$ cards, on each of which is written a positive integer. The pack is shuffled and the cards laid out in a row, with the numbers facing upwards. Alice starts, and the girls take turns to remove one card from either end of the row, until Barbara picks up the final card. Each girl's score is the sum of the numbers on her chosen cards at the end of the game.
Prove that Alice can always obtain a score at least as great as Barbara's.

4. A set of positive integers is defined to be *wicked* if it contains no three consecutive integers. We count the empty set, which contains no elements at all, as a wicked set.
Find the number of wicked subsets of the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

5. Let p, q and r be prime numbers. It is given that p divides $qr - 1$, q divides $rp - 1$, and r divides $pq - 1$.
Determine all possible values of pqr.